Solution

Mech 311 Thermodynamics 2 — **November 8, 2006**
Quiz 1 — **90 minutes**

Hello people. Please read the questions carefully. As you solve them, indicate clearly your system/CV boundaries and any assumptions you might make. When possible, please state in words what you are trying to solve/your approach so that I can follow your work and give you credit. This is especially important if you run out of time. Most importantly, relax!

SHORT ANSWER (5 points)

1. What guarantees a net power output from the ideal Rankine cycle? (2 points)
2. How effective would a vapor compression cycle (compressor–condenser–throttle valve–evaporator) be if the working fluid in it was an ideal gas? (3 points)

A DUAL-LOOP HEAT-POWERED REFRIGERATION CYCLE

To take advantage of a source of waste heat to cool a space, a gas turbine is modified to make a dual-loop heat-powered refrigeration cycle using Freon 12 as the working fluid, as shown below. Saturated vapor at 105 C leaves the boiler and expands reversibly in the adiabatic turbine to the condenser pressure. Saturated vapor at -15 C leaves the evaporator and is reversibly and adiabatically compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two existing streams mix together and enter the condenser. Saturated liquid at 45 C leaving the condenser is then separated into two streams in the necessary proportions. Pressure drops in all the piping are negligible. (NOTE: some useful data on superheated R12 is given below)

Part 1 (60 points)

Determine the following:

a) The enthalpy at point 3 (15 points)
b) The enthalpy at point 8 (10 points)
c) $\dot{m}_1 / \dot{m}_2$, the ratio of mass flow through the power loop to that through the refrigeration loop (15 points)
d) The coefficient of performance (COP), defined as the cooling effect, Q_L, divided by the sum of heat and power input to the power loop. (10 points)
e) Calculate the COP if the power loop were replaced by an electric motor, all else remaining the same (10 points)

Part 2 (35 points)

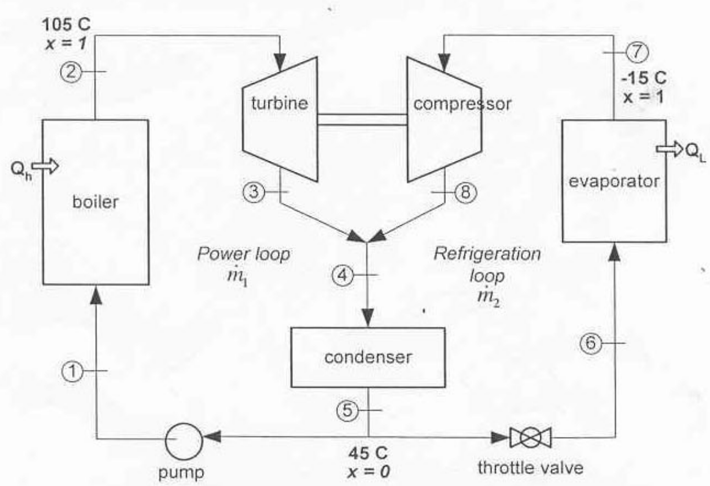
a) Explain why the COP of this system is considerably lower than a value of about 3 for common refrigeration systems available on the market. (10 points)
b) The power loop produces relatively little power per unit of heat input. Why? Would you recommend replacing the power loop with an electric motor? (10 points)
c) There are only two internal irreversibilities in this dual-loop system. Identify them and calculate for each the exergy destroyed per unit mass flow $\dot{m}_2$. Assume T_0 = 30 deg C. (15 points)

EXTRA CREDIT (10 points)

Carefully sketch a T-s diagram of the entire system; on the T-axis, indicate -15, 45, and 105 C and keep those temperatures in mind as you draw the diagram.

Superheated R12 data
for P = 1.084 MPa

T (C)	s(kJ/kg-K)	h (kJ/kg)
50	0.6937	208.8
60	0.7176	216.6
70	0.7403	224.3



Quiz 1 Solution Fall 2006-7

Short Answer

1. $W_{net} = W_{turbine} - W_{pump}$

if both are reversible steady devices operating adiabatically, then $\Delta s = 0$. Gibbs law dictates that $W_{rev,ad.} = \int v\,dP$

+1 $\Rightarrow W_{net} = \int_{(t)} v\,dP - \int_{(P)} v\,dP$; since ΔP is same for P & t

then $W_{net} = \int (v_t - v_p)\,dP$ but $v_t >> v_p \Rightarrow W_{net} > 0$ +1

2. it would not work. since Δh across the throttle valve is zero, and for an ideal gas $\Delta h = c_p \Delta T$ then $\Delta T = 0$ across the valve. $\Rightarrow T_{condenser} = T_{evap}$ and no heat is moved from either space. (it may actually heat both spaces)

Part I.

+7 a) $h_3 = h(s_2, P_3)$

+5 but $P_3 = P_4 = P_5 = P_{sat}(45°C)$

+3 $s_2 = s_g(105°C)$

$h_3 = 189.2$ kJ/kg

+7 b) $h_8 = h(s_8, P_8) = h(s_7, P_5)$

+3 $s_7 = s_g(-15°C)$

$\Rightarrow h_8 = 212.3$ kJ/kg

+5 c) $W_t = -W_c$

+6 $\dot{m}_1 (h_2 - h_3) = \dot{m}_2 (h_8 - h_7)$

$$\frac{\dot{m}_1}{\dot{m}_2} = \frac{h_8 - h_7}{h_2 - h_3}$$

where h_8 = given above

+2 $h_7 = h_g(-15°C)$

+2 $h_2 = h_g(105°C)$

h_3 = given above

$\dot{m}_1 / \dot{m}_2 = 1.87$

USE FOR ANSWER

7/12

+5 +3

d) $COP = \frac{Q_L}{W_c + P_{pump}} = \frac{\dot{m}_2 (h_7 - h_6)}{\dot{m}_1 (h_2 - h_5)}$ (COP = 0.43)

where h_7 = given above

+1 $h_6 = h_5$

h_2 = given above

+1 $h_5 = h_f (45°C)$

$\dot{m}_2/\dot{m}_1$ = given above

e) if replaced by an electric motor

$COP = \frac{Q_L}{\dot{W}_c} = \frac{h_7 - h_6}{h_8 - h_7} = 3.2$

+7 +3

Part II

a) The COP of this system is low because of the power loop. We know this from Part 1c, where we found that removing the power loop restored the COP to a typical value of about 3. COPs for refrigerators are always reported without taking into account the efficiency of electric power generation and transmission. If they did, the COP would be less than 1.

b) The power loop runs at low efficiency because the difference in temperature at which heat addition occurs in the boiler and heat rejection occurs in the condenser is very small. This is to be expected because we are using a low-value waste heat source. We would NOT prefer to pay for electricity when waste heat is available. Remember that the refrigeration loop will perform exactly the same whether the compressor work comes from a turbine or an electric motor!

c) the two irreversibilities are at the mixer (streams 3+8 → 4) and the throttle valve

exergy destroyed = $T_0 S_{gen}$

<u>for mixer:</u>

+4
$$S_{gen} = \dot{m}_4 s_4 - (\dot{m}_3 s_3 + \dot{m}_8 s_8)$$
$$= (\dot{m}_1 + \dot{m}_2) s_4 - \dot{m}_1 s_3 - \dot{m}_2 s_8$$
$$\frac{S_{gen}}{\dot{m}_2} = \left(\frac{\dot{m}_1}{\dot{m}_2} + 1\right) s_4 - \frac{\dot{m}_1}{\dot{m}_2} s_3 - s_8$$

+1 $s_4 = s(h_4, P_4)$

+1 where $P_4 = P_{sat}(45°C)$

+3
$$h_4 = \frac{\dot{m}_1 h_3 + \dot{m}_2 h_8}{\dot{m}_1 + \dot{m}_2} \quad (1^{st} \text{ law for mixer})$$
$$= \frac{(\dot{m}_1/\dot{m}_2) h_3 + h_8}{\left(\frac{\dot{m}_1}{\dot{m}_2} + 1\right)}$$
$$= 197.3$$

+1
$$\Rightarrow x_4 = 0.94$$
$$\Rightarrow s_4 = 0.66$$

after substitution of numerical values we find $S_{gen} \approx 0$ (within the accuracy of the calculation) for the valve

+2 $\frac{S_{gen}}{\dot{m}_2} = s_6 - s_5$

+1 $s_6 = s(h_6, P_6)$ $s_5 = s_f(45°C)$

$P_6 = P_{sat}(-15°C)$

$h_6 = h_5$

+2 $\Rightarrow x_6 = 0.36$

$\Rightarrow s_6 = 0.3127$

$\frac{S_{gen}}{\dot{m}_2} = 0.0252 \Rightarrow \frac{E_{xd}}{\dot{m}} = (0.0252)(273+30) = 7.64$ kJ

T-s diagram